

Concealed Quantum Information

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We study the teleportation scheme performed by means of a partially entangled pure state. We found that the information belonging to the quantum channel can be distributed into both the system of the transmitter and the system of the receiver. Thus, in order to complete the teleportation process it is required to perform an *unambiguous non-orthogonal quantum states discrimination* and an *extraction of the quantum information* processes. This general scheme allows one to design a strategy for concealing the unknown information of the teleported state. Besides, we showed that the *teleportation* and the *concealing the quantum information* process, can be probabilistically performed even though the bipartite quantum channel is maximally entangled.

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Quantum teleportation is a protocol which allows transferring unknown information codified in a pure state, from one quantum system to another similar one [1]. In order to teleport a pure state being in a two dimensional Hilbert space, one requires a Bell state (ebit) and two units with classical information (2 bits). These resources, an ebit and two bits, complement each other in the sense that together they allow transferring two unknown real numbers, codified in a quantum state, between two remote places. When the quantum channel is a partially entangled pure state, the process of teleporting an unknown state becomes probabilistic. The probabilistic teleportation procedure can be implemented by the transmitter [2, 3] or by the receiver [4, 5]. Throughout this article we will call *channel information* the probability amplitude of the bipartite entangled state, $|\phi^+\rangle_{a_2b}$, shared by the transmitter and the receiver, i.e., $\cos(\theta/2)$ and $\sin(\theta/2)$, see Eq. (2). In the first probabilistic teleportation scheme the channel information is completely assumed by the transmitter who must perform an *unambiguous non-orthogonal quantum states discrimination* (UQSD) process [6, 7] in order to complete the process. In the second scheme the channel information is completely assumed by the receiver who must perform an *extraction of the quantum information* (EQI) process [4] in order to complete the quantum teleportation procedure.

In this article we show that the channel information can be shared by both the system of the transmitter and by the system of the receiver. One can realize how this scheme allows designing a strategy to hide the unknown information codified in the state to be teleported. Since this process called *concealing the quantum information* is not unitary and the hidden information is unknown, it can not be retrieved by means of a unitary process but only by means of a probabilistic procedure. Besides, we stress the fact found here that both, the teleportation and the *concealing the quantum information* process, can be probabilistically performed even though the bipartite quantum channel is maximally entangled.

First of all, we review the reported probabilistic tele-

portation schemes for the particular case of two dimensional Hilbert spaces. We denote by subindexes a_1 and a_2 the systems belonging to the transmitter (laboratory a) and by the subindex b the receiving system. The unknown state, $|\psi\rangle_{a_1}$, to be teleported is codified on the a_1 system. The a_2 and b systems are previously prepared in the partially entangled state $|\tilde{\phi}^+\rangle_{a_2b}$, see Eq. (2).

A probabilistic teleportation scheme was reported in Refs. [2, 3] and it can be succinctly described by the following identity:

$$|\psi\rangle_{a_1}|\tilde{\phi}^+\rangle_{a_2b} = \frac{1}{2} \left[|\tilde{\psi}^+\rangle_a \sigma_x |\psi\rangle_b - |\tilde{\psi}^-\rangle_a \sigma_x \sigma_z |\psi\rangle_b + |\tilde{\phi}^+\rangle_a |\psi\rangle_b - |\tilde{\phi}^-\rangle_a \sigma_z |\psi\rangle_b \right], \quad (1)$$

where the four linear independent bipartite states are given by:

$$\begin{aligned} |\tilde{\psi}^\pm\rangle_a &= \sin \frac{\theta}{2} |0\rangle_{a_1} |1\rangle_{a_2} \pm \cos \frac{\theta}{2} |1\rangle_{a_1} |0\rangle_{a_2}, \\ |\tilde{\phi}^\pm\rangle_a &= \cos \frac{\theta}{2} |0\rangle_{a_1} |0\rangle_{a_2} \pm \sin \frac{\theta}{2} |1\rangle_{a_1} |1\rangle_{a_2}. \end{aligned} \quad (2)$$

Without loss of generality we assume $0 \leq \theta \leq \pi/2$. We have written the Pauli operators as $\sigma_x = |0\rangle\langle 1| + |1\rangle\langle 0|$, $\sigma_z = |1\rangle\langle 1| - |0\rangle\langle 0|$ where evidently $\{|0\rangle, |1\rangle\}$ is the eigenbasis of σ_z . So, by carrying out a UQSD procedure on the (2) states, which has probability of success

$$p_s = 1 - \cos \theta, \quad (3)$$

and transmitting the results (classical information), the receiver could successfully complete the teleportation process by performing the appropriate unitary operator. In other words, the process has probability p_s of teleporting the unknown state $|\psi\rangle$ with fidelity 1.

Another probabilistic teleportation scheme was first proposed in Ref. [4]. It is extended to the d -dimensional Hilbert space case by Li-YI Hsu [5]. That scheme can be

understood from the following expansion:

$$|\psi\rangle_{a_1}|\tilde{\phi}^+\rangle_{a_2,b} = \sqrt{p}\frac{|\psi^+\rangle_a\sigma_x|\tilde{\psi}\rangle_b - |\psi^-\rangle_a\sigma_x\sigma_z|\tilde{\psi}\rangle_b}{\sqrt{2}} + \sqrt{1-p}\frac{|\phi^+\rangle_a|\hat{\psi}\rangle_b - |\phi^-\rangle_a\sigma_z|\hat{\psi}\rangle_b}{\sqrt{2}}, \quad (4)$$

where $|\phi^\pm\rangle_a$ and $|\psi^\pm\rangle_a$ are the Bell states, i.e., they are the states of Eq. (2) evaluated with $\theta = \pi/2$. The p probability is read as

$$p = |\langle 0|\psi\rangle|^2 \sin^2(\theta/2) + |\langle 1|\psi\rangle|^2 \cos^2(\theta/2). \quad (5)$$

Thus, after applying a measurement on the Bell basis of the $a_1 \oplus a_2$ bipartite system and communicating these results to the receiver, the state of the b system becomes

$$|\tilde{\psi}\rangle_b = \frac{1}{\sqrt{p}}(\langle 0|\psi\rangle \sin \frac{\theta}{2}|0\rangle_b + \langle 1|\psi\rangle \cos \frac{\theta}{2}|1\rangle_b), \quad (6)$$

or

$$|\hat{\psi}\rangle_b = \frac{1}{\sqrt{1-p}}(\langle 0|\psi\rangle \cos \frac{\theta}{2}|0\rangle_b + \langle 1|\psi\rangle \sin \frac{\theta}{2}|1\rangle_b). \quad (7)$$

In order to complete the teleportation, the receiver must apply an EQI process on the (7) or on the (6) state. For instance, with probability p the outcome state is (6). In this case the receiver must apply a *Control-U* unitary operator [8], $\tilde{\chi}_{bB}$, with system b being the control and an auxiliary system B , prepared in the $|0\rangle_B$ state, being the target; Namely:

$$\tilde{\chi}_{bB} = |0\rangle_{bb}\langle 0| \otimes I_B + |1\rangle_{bb}\langle 1| \otimes e^{i\sigma_y \arccos(\tan \frac{\theta}{2})}, \quad (8)$$

with $\sigma_y = -i\sigma_z\sigma_x$, and I_B being the identity of the Hilbert space of the B system. It is understood that $e^{i\sigma_y \arccos(\tan \frac{\theta}{2})}$ operates on the Hilbert space of the B system. Since

$$\tilde{\chi}_{bB}|\tilde{\psi}\rangle_b|0\rangle_B = \frac{\sin \frac{\theta}{2}}{\sqrt{p}}|\psi\rangle_b|0\rangle_B + \frac{\langle 1|\psi\rangle\sqrt{\cos \theta}}{\sqrt{p}}|1\rangle_b|1\rangle_B,$$

a measurement process performed on the auxiliary system allows extracting the quantum information, $|\psi\rangle_b$,

with probability $\sin^2(\theta/2)$. Similarly, with probability $1-p$ the outcome is the (7) state; then the receiver must apply, on the $|\hat{\psi}\rangle_b|0\rangle_B$ state, the transformed unitary *Control-U* gate ($e^{i\sigma_x\pi/2} \otimes I_B$) $\tilde{\chi}_{b_1B}(e^{-i\sigma_x\pi/2} \otimes I_B)$, where it is understood that $e^{\pm i\sigma_x\pi/2}$ takes action on the Hilbert space of the b system. In this case the probability of *extracting the quantum information* is also $\sin^2(\theta/2)$. Thus, the whole success probability in the EQI procedure is $2\sin^2(\theta/2)$ which is just the probability (3). Therefore both processes, the teleportation completed by a UQSD (T-UQSD) process and the teleportation completed by an EQI (T-EQI) process have the same optimal probability of success. However, a difference between these schemes is that in the T-EQI process an extra auxiliary system is required as a physical resource. Nevertheless it must be emphasized that in the T-UQSD process the transmitter needs an extra bit for informing the success or the failure of the measurement process.

Now we show how the teleportation scheme which requires both the UQSD and the EQI protocols in order to be completed can be used to design a strategy to conceal unknown quantum information.

The above described processes, T-UQSD and T-EQI, are based on two different decompositions of the $|\psi\rangle_{a_1}|\phi^+\rangle_{a_2,b}$ tripartite state, say, on the four nonorthogonal linear independent states, (2), or on the orthonormal Bell basis. Here we start our analysis by considering the expansion of the $|\psi\rangle_{a_1}|\phi^+\rangle_{a_2,b}$ state on the general nonorthogonal bipartite basis

$$\begin{aligned} |\tilde{\phi}_{xy}^\pm\rangle_a &= \frac{\cos^x \frac{\theta}{2}|0\rangle_{a_1}|0\rangle_{a_2} \pm \sin^y \frac{\theta}{2}|1\rangle_{a_1}|1\rangle_{a_2}}{\sqrt{\cos^{2x} \frac{\theta}{2} + \sin^{2y} \frac{\theta}{2}}}, \\ |\tilde{\psi}_{xy}^\pm\rangle_a &= \frac{\sin^y \frac{\theta}{2}|0\rangle_{a_1}|1\rangle_{a_2} \pm \cos^x \frac{\theta}{2}|1\rangle_{a_1}|0\rangle_{a_2}}{\sqrt{\cos^{2x} \frac{\theta}{2} + \sin^{2y} \frac{\theta}{2}}}, \end{aligned} \quad (9)$$

where the two real parameters x and y go independently from 0 to 1. Thus, we obtain the identity

$$|\psi\rangle_{a_1}|\tilde{\phi}\rangle_{a_2,b} = \sqrt{p}\frac{|\tilde{\psi}_{xy}^+\rangle_a\sigma_x|\tilde{\psi}_{xy}\rangle_b - |\tilde{\psi}_{xy}^-\rangle_a\sigma_x\sigma_z|\tilde{\psi}_{xy}\rangle_b}{\sqrt{\mu}} + \sqrt{1-p}\frac{|\tilde{\phi}_{xy}^+\rangle_a|\hat{\psi}_{xy}\rangle_b - |\tilde{\phi}_{xy}^-\rangle_a\sigma_z|\hat{\psi}_{xy}\rangle_b}{\sqrt{\nu}}, \quad (10)$$

where the normalization constants are

$$\begin{aligned} \mu &= \frac{4p \left(\cos^{2x} \frac{\theta}{2} + \sin^{2y} \frac{\theta}{2} \right)^{-1}}{\left(|\langle 0|\psi\rangle|^2 \sin^{2(1-y)} \frac{\theta}{2} + |\langle 1|\psi\rangle|^2 \cos^{2(1-x)} \frac{\theta}{2} \right)}, \\ \nu &= \frac{4(1-p) \left(\cos^{2x} \frac{\theta}{2} + \sin^{2y} \frac{\theta}{2} \right)^{-1}}{\left(|\langle 0|\psi\rangle|^2 \cos^{2(1-x)} \frac{\theta}{2} + |\langle 1|\psi\rangle|^2 \sin^{2(1-y)} \frac{\theta}{2} \right)}, \end{aligned}$$

and the outcome states are read as

$$|\tilde{\psi}_{xy}\rangle_b = \frac{\langle 0|\psi\rangle \sin^{1-y} \frac{\theta}{2}|0\rangle_b + \langle 1|\psi\rangle \cos^{1-x} \frac{\theta}{2}|1\rangle_b}{\sqrt{|\langle 0|\psi\rangle|^2 \sin^{2(1-y)} \frac{\theta}{2} + |\langle 1|\psi\rangle|^2 \cos^{2(1-x)} \frac{\theta}{2}}} \quad (11)$$

$$|\hat{\psi}_{xy}\rangle_b = \frac{\langle 0|\psi\rangle \cos^{1-x} \frac{\theta}{2}|0\rangle_b + \langle 1|\psi\rangle \sin^{1-y} \frac{\theta}{2}|1\rangle_b}{\sqrt{|\langle 0|\psi\rangle|^2 \cos^{2(1-x)} \frac{\theta}{2} + |\langle 1|\psi\rangle|^2 \sin^{2(1-y)} \frac{\theta}{2}}} \quad (12)$$

From Eqs. (10), (9), and (12) we can notice that, if the transmitter defines the parameters x and y to perform an UQSD process on the $|\phi_{xy}^\pm\rangle_a$ and $|\psi_{xy}^\pm\rangle_a$ states, then the outcome state, Eq. (12), of the b system gets partial information of the channel. In this form, the unknown information of the $|\psi\rangle$ state is concealed by means of the two real parameters x and y . We call this protocol *concealing the quantum information* (CQI). Since the CQI procedure is probabilistic and the $|\psi\rangle$ state is unknown, it can only be probabilistically retrieved, performing an appropriate EQI process only by the party who knows the x and y parameters. We suppose that the parameter θ could be publicly known.

We also notice that in the particular case $x = y = 0$ the T-EQI scheme arises, whereas in the particular case $x = y = 1$ the T-UQSD protocol holds. It is worth emphasizing that, when the quantum channel is maximally entangled, i.e., $\theta = \pi/2$, the teleportation procedure is deterministic only in the particular cases $x = y$; otherwise the teleportation process is probabilistic. This result is counterintuitive since one would think that, with maximally entangled states, the process is always deterministic [1, 2, 3, 4, 5]. Therefore, even in the case of a maximally entangled channel, the information, $|\psi\rangle$, can be concealed by choosing two different x and y parameters.

The probability of discriminating conclusively among the four linear independent non-orthogonal states (9) is

$$\begin{aligned} p_{UQSD} &= p(1 - |\langle\tilde{\psi}_{xy}^-|\tilde{\psi}_{xy}^+\rangle|) + (1-p)(1 - |\langle\tilde{\phi}_{xy}^-|\tilde{\phi}_{xy}^+\rangle|), \\ &= 1 - \frac{|\cos^{2x}\frac{\theta}{2} - \sin^{2y}\frac{\theta}{2}|}{\cos^{2x}\frac{\theta}{2} + \sin^{2y}\frac{\theta}{2}}. \end{aligned} \quad (13)$$

This probability, Eq. (13), corresponds to the probability of concealing the quantum information. Its maximum value 1 happens for $\cos^x\frac{\theta}{2} = \sin^y\frac{\theta}{2}$, in this case the (9) states become the Bell states and the outcome states (12) become the (6) and (7) states. For a given θ , the minimum value of $p_{UQSD} = 2\sin^2(\theta/2)/[1 + \sin^2(\theta/2)]$ corresponding to $x = 0$ and $y = 1$. In other words,

$\min\{p_{UQSD}\}_{\{x,y\}}$ is the infimum probability for the process of concealing the quantum information.

As we already see, the probability of concealing the unknown quantum information, $|\psi\rangle$, by the secret parameters x and y is equal to the probability of discriminating conclusively among the linear independent non-orthogonal states (9) and, since the probabilities of retrieving the quantum information are equal to the probabilities of extracting the quantum information from the states (11) and (12), i.e.,

$$\check{p}_{EQI} = \frac{\min\{\sin^{2(1-y)}\frac{\theta}{2}, \cos^{2(1-x)}\frac{\theta}{2}\}}{|\langle 0|\psi\rangle|^2 \sin^{2(1-y)}\frac{\theta}{2} + |\langle 1|\psi\rangle|^2 \cos^{2(1-x)}\frac{\theta}{2}}, \quad (14)$$

$$\hat{p}_{EQI} = \frac{\min\{\sin^{2(1-y)}\frac{\theta}{2}, \cos^{2(1-x)}\frac{\theta}{2}\}}{|\langle 0|\psi\rangle|^2 \cos^{2(1-x)}\frac{\theta}{2} + |\langle 1|\psi\rangle|^2 \sin^{2(1-y)}\frac{\theta}{2}}, \quad (15)$$

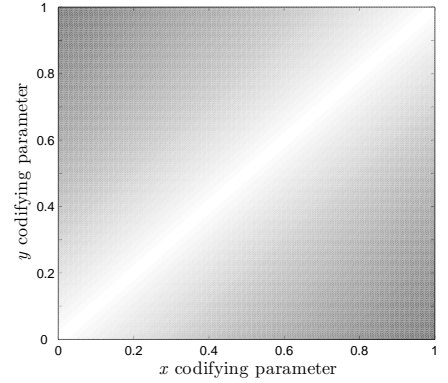


FIG. 1: Linear black-white degradation of the probability p_{xy} as a function of the x and y secret dimensionless codifying parameters for; $\theta = \pi/2$ and $|\psi\rangle = (|0\rangle + \sqrt{2}|1\rangle)/\sqrt{3}$. White color means probability 1 whereas the darkest color stands for its minimum probability value 0.45.

the whole probability of success, of the process of teleporting unknown concealed information and thence retrieve it, is given by

$$\begin{aligned} p_{xy} &= p(1 - |\langle\tilde{\psi}_{xy}^-|\tilde{\psi}_{xy}^+\rangle|)\check{p}_{EQI} + (1-p)(1 - |\langle\tilde{\phi}_{xy}^-|\tilde{\phi}_{xy}^+\rangle|)\hat{p}_{EQI}, \\ &= \frac{2 \min\{\cos^{2x}\frac{\theta}{2}, \sin^{2y}\frac{\theta}{2}\} \min\{\cos^{2(1-x)}\frac{\theta}{2}, \sin^{2(1-y)}\frac{\theta}{2}\}}{\cos^{2x}\frac{\theta}{2} + \sin^{2y}\frac{\theta}{2}} \\ &\quad \times \left(\frac{|\langle 0|\psi\rangle|^2 \cos^2\frac{\theta}{2} + |\langle 1|\psi\rangle|^2 \sin^2\frac{\theta}{2}}{|\langle 0|\psi\rangle|^2 \cos^{2(1-x)}\frac{\theta}{2} + |\langle 1|\psi\rangle|^2 \sin^{2(1-y)}\frac{\theta}{2}} + \frac{|\langle 0|\psi\rangle|^2 \sin^2\frac{\theta}{2} + |\langle 1|\psi\rangle|^2 \cos^2\frac{\theta}{2}}{|\langle 0|\psi\rangle|^2 \sin^{2(1-y)}\frac{\theta}{2} + |\langle 1|\psi\rangle|^2 \cos^{2(1-x)}\frac{\theta}{2}} \right). \end{aligned} \quad (16)$$

We can notice that: the p_{xy} probability depends on the state to be teleported, when $x = y = 0$ or $x = y = 1$ the probability p_{xy} is equal to p_s . We can also notice that

the T-UQSD and T-EQS processes can be obtained when $\cos^x(\theta/2) = \sin^y(\theta/2)$ and $\cos^{(1-x)}(\theta/2) = \sin^{(1-y)}(\theta/2)$ respectively. Figure 1 shows a linear black-white degra-

dation of the probability p_{xy} as a function of the x and y dimensionless codifying parameters for; $\theta = \pi/2$ and $|\psi\rangle = (|0\rangle + \sqrt{2}|1\rangle)/\sqrt{3}$. White color means probability 1 whereas darkest color stands for the minimum probability value 0.45. From Fig. 1 clearly we notice that, in this case of maximally entangled channel, the p_{xy} probability is 1 on the diagonal $x = y$ only; for other values of the (x, y) the process is probabilistic and its minimum probability is different from zero.

In summary, we have studied a general teleportation process which combines a *conclusive non-orthogonal quantum states discrimination* process with an *extraction of the quantum information* scheme. In this form we find a new non-unitary strategy which allows concealing the quantum information by means of two real parameters which can be secretly chosen by the party who wants to protect or hide the information. Since the process of concealing the unknown quantum information is non-unitary, the process of retrieving it can not be unitary but it can be probabilistic. Besides, we showed that both, the teleportation and the *concealing the quantum information* processes, can be probabilistically performed even though the bipartite quantum channel is maximally entangled.

Currently the *deterministic quantum teleportation* scheme has been experimentally performed with twin-photons and single photons [9, 10, 11, 12, 13] and with cold ions $^{40}\text{Ca}^+$ and $^9\text{Be}^+$ moving in a linear Paul trap [14, 15] There is also a proposition for teletransporting an atomic state between two cavities using nonlocal microwave fields [16]. On the other hand, the *unambiguous nonorthogonal quantum states discrimination* scheme has been experimentally demonstrated for two nonorthogonal states of light [17] and also it has been theoretically proposed with cold ions in a linear Paul trap [7]. In this way, an experimental physical implementation of the here proposed *concealing the quantum information* protocol could consider systems such as single photons or cold ions.

Further studies of the above described *concealing the quantum information* scheme can be realized by considering d-dimensional Hilbert spaces.

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